MICROMECHANICS OF METAL MATRIX COMPOSITES USING THE GENERALIZED METHOD OF CELLS MODEL (GMC) USER'S GUIDE

by

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ABSTRACT

A user's guide for the program gmc.f is presented. The program is based on the generalized method of cells model (GMC), which is capable via a micromechanical analysis, of predicting the overall, inelastic behavior of unidirectional, multi-phases composites from the knowledge of the properties of the viscoplastic constituents. In particular, the program is sufficiently general to predict the response of undirectional composites having variable fiber shapes and different fiber arrays.

INTRODUCTION

The method of cells is a micromechanical model which has been shown to accurately predict the overall behavior of various types of composites from the knowledge of the constituent properties. In particular, the method yields explicit effective constitutive equations for the inelastic behavior of metal matrix composites. The various capabilities of the method were demonstrated in a recent review paper and a monograph [1,2].

The overall behavior of inelastic, multi-phase, unidirectional fibrous composites generated by the method of cells from the knowledge of the properties of the individual constituents is displayed in terms of:

- effective elastic moduli

- effective coefficients of thermal expansion

- effective thermal conductivities

- effective stress-strain response in the inelastic region

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Original method of cells

In the original formulation of the method of cells, a continously-reinforced, unidirectional fibrous composite is modeled as a doubly-periodic array of fibers embedded in a matrix phase, Figure 1a. The periodic character of the assemblage allows one to identify a repeating unit cell that can be used as a building block to construct the entire composite, Figure 1b. The properties of this repeating unit cell are thus representative of the properties of the entire assemblage. The unit cell consists of a single fiber subcell surrounded by three matrix subcells. Hence the name **method of cells**. The rectangular geometry of the repeating unit cell allows one to obtain an approximate solution for the stresses and strains in the individual subcells given some macroscopically homogeneous state of strain or stress applied to the composite. The approximate solution to the thus posed boundary-value problem is, in turn, used to determine macroscopic (average) or effective properties of the composite.

The displacement field in the individual subcells is approximated in terms of a linear expansion in the local coordinates $\bar{x}^{(\beta)}$, $\bar{x}^{(\gamma)}$ centered at the mid-point of a given subcell,

$$u_i^{(\beta\gamma)} = \omega_i^{(\beta\gamma)} + \bar{x}_2^{(\beta)}\phi_i^{(\beta\gamma)} + \bar{x}_3^{(\gamma)}\psi_i^{(\beta\gamma)} \ , \qquad\qquad i = 1,2,3 \qquad\qquad (1)$$

where $\omega_i^{(\beta\gamma)}$ are the displacement components at the center of each subcell $(\beta\gamma)$. Using the local strain-displacement equations for each subcell, one can easily show that the microvariables $\phi_i^{(\beta\gamma)}$ and $\psi_i^{(\beta\gamma)}$ are related to the subcell strains $\epsilon_{ij}^{(\beta\gamma)}$. For example,

$$\epsilon_{11}^{(\beta\gamma)} = \frac{\partial\omega_1^{(\beta\gamma)}}{\partial x_1} \ , \qquad \epsilon_{22}^{(\beta\gamma)} = \phi_2^{(\beta\gamma)} \ , \qquad \epsilon_{33}^{(\beta\gamma)} = \psi_3^{(\beta\gamma)} \ , \qquad\qquad (2)$$

with the shear strains given by similar expressions.

The displacement expansion given by Equation (1) contains 9 unknown variables in each subcell of the repeating unit cell, i.e., three $\omega_i^{(\beta\gamma)}$'s, three $\phi_i^{(\beta\gamma)}$'s and three $\psi_i^{(\beta\gamma)}$'s, for a total of 36 unknown variables per unit cell. These unknown variables are expressed in terms of the macroscopic or average strains applied to the composite by imposing the continuity of displacements between the individual subcells, as well as between the given repeating cell and the surrounding cells, together with the continuity of interfacial tractions. These continuity conditions are applied in an average sense due to the form of the diplacement representation in the individual subcells. Application of these continuity conditions leads to the following set of equations:

Continuity of interfacial displacements

$$h_1 \phi_i^{(1\gamma)} + h_2 \phi_i^{(2\gamma)} = (h_1 + h_2) \frac{\partial \omega_i}{\partial x_2}$$

$$l_1 \psi_i^{(\beta 1)} + l_2 \psi_i^{(\beta 2)} = (l_1 + l_2) \frac{\partial \omega_i}{\partial x_3} \tag{3}$$

$$\bar{\varepsilon}_{ij} = \frac{1}{2} (\frac{\partial \omega_i}{\partial x_j} + \frac{\partial \omega_j}{\partial x_i})$$

where $\omega_i^{(11)} = \omega_i^{(12)} = \omega_i^{(21)} = \omega_i^{(22)} = \omega_i$.

Continuity of interfacial tractions

$$\bar{\sigma}_{2i}^{(1\gamma)} = \bar{\sigma}_{2i}^{(2\gamma)} \tag{4}$$

$$\bar{\sigma}_{3i}^{(\beta 1)} = \bar{\sigma}_{3i}^{(\beta 2)} \tag{5}$$

Equations (3)-(5), together with the appropriate constitutive equations for the given subcell $(\beta\gamma)$, are used to express the microvariables in terms of the macroscopic strains, $\bar{\varepsilon}_{ij}$, applied to the composite. The form of the constitutive equations used to describe the material behavior in the subcell $(\beta\gamma)$ is,

$$\bar{\sigma}^{(\beta\gamma)} = C^{(\beta\gamma)} [\bar{\varepsilon}^{(\beta\gamma)} - \bar{\varepsilon}^{p(\beta\gamma)} - \alpha^{(\beta\gamma)} \Delta T] \tag{6}$$

where $\bar{\sigma}^{(\beta\gamma)}$ is the average stress, $\bar{\varepsilon}^{(\beta\gamma)}$ is the average total strain, $\bar{\varepsilon}^{p(\beta\gamma)}$ is the average plastic strain, $C^{(\beta\gamma)}$ is the elastic stiffness matrix, $\alpha^{(\beta\gamma)}$ are the coefficients of thermal expansion, and ΔT is the temperature deviation of the material occupying subcell $(\beta\gamma)$. The volume averaged subcell stresses $\bar{\sigma}^{(\beta\gamma)}$ are defined in the usual way,

$$\bar{\sigma}^{(\beta\gamma)} = \frac{1}{V_{\beta\gamma}} \int_{-h_\beta/2}^{+h_\beta/2} \int_{-l_\gamma/2}^{+l_\gamma/2} \sigma^{(\beta\gamma)} d\bar{x}_2^{(\beta)} d\bar{x}_3^{(\gamma)} \tag{7}$$

(where $V_{\beta\gamma} = h_\beta l_\gamma$), with similar definitions for the remaining averaged quantities. The volume averaged subcell stresses are obtained in terms of the microvariables in the following form:

$$\bar{\sigma}_{11}^{(\beta\gamma)} = C_{11}^{(\beta\gamma)}(\bar{\varepsilon}_{11} - \bar{\varepsilon}_{11}^{p(\beta\gamma)} - \alpha_{11}^{(\beta\gamma)}\Delta T) + C_{12}^{(\beta\gamma)}[(\phi_2^{(\beta\gamma)} - \bar{\varepsilon}_{22}^{p(\beta\gamma)} - \alpha_{22}^{(\beta\gamma)}\Delta T) + (\psi_3^{(\beta\gamma)} - \bar{\varepsilon}_{22}^{p(\beta\gamma)} - \alpha_{33}^{(\beta\gamma)}\Delta T)]$$

$$\bar{\sigma}_{22}^{(\beta\gamma)} = C_{12}^{(\beta\gamma)}(\bar{\varepsilon}_{11} - \bar{\varepsilon}_{11}^{p(\beta\gamma)} - \alpha_{11}^{(\beta\gamma)}\Delta T) + C_{11}^{(\beta\gamma)}(\phi_2^{(\beta\gamma)} - \bar{\varepsilon}_{22}^{p(\beta\gamma)} - \alpha_{22}^{(\beta\gamma)}\Delta T) + C_{12}^{(\beta\gamma)}(\psi_3^{(\beta\gamma)} - \bar{\varepsilon}_{22}^{p(\beta\gamma)} - \alpha_{33}^{(\beta\gamma)}\Delta T)$$

$$\bar{\sigma}_{22}^{(\beta\gamma)} = C_{12}^{(\beta\gamma)}(\bar{\varepsilon}_{11} - \bar{\varepsilon}_{11}^{p(\beta\gamma)} - \alpha_{11}^{(\beta\gamma)}\Delta T) + C_{12}^{(\beta\gamma)}(\phi_2^{(\beta\gamma)} - \bar{\varepsilon}_{22}^{p(\beta\gamma)} - \alpha_{22}^{(\beta\gamma)}\Delta T) + C_{11}^{(\beta\gamma)}(\psi_3^{(\beta\gamma)} - \bar{\varepsilon}_{22}^{p(\beta\gamma)} - \alpha_{33}^{(\beta\gamma)}\Delta T)$$

$$\bar{\sigma}_{23}^{(\beta\gamma)} = C_{66}^{(\beta\gamma)}(\psi_2^{(\beta\gamma)} + \phi_3^{(\beta\gamma)} - \bar{\varepsilon}_{23}^{p(\beta\gamma)})$$

$$\bar{\sigma}_{13}^{(\beta\gamma)} = C_{66}^{(\beta\gamma)}(\psi_1^{(\beta\gamma)} + \frac{\partial\omega_3}{\partial x_1} - \bar{\varepsilon}_{13}^{p(\beta\gamma)})$$

$$\bar{\sigma}_{12}^{(\beta\gamma)} = C_{66}^{(\beta\gamma)}(\phi_1^{(\beta\gamma)} + \frac{\partial\omega_2}{\partial x_1} - \bar{\varepsilon}_{12}^{p(\beta\gamma)})$$

The above formulation is sufficiently general to admit any inelastic constitutive model for the average plastic strains $\bar{\varepsilon}_{ij}^{(\beta\gamma)}$ in the subcell $(\beta\gamma)$. Presently, the Bodner-Partom unified viscoplasticity theory is used to model the inelastic effects. This constitutive model will be briefly described in the following section.

The effective or average stress-strain equations for the composite are subsequently constructed by applying the definition of the average composite stresses:

$$\bar{\sigma}_{ij} = \frac{1}{V} \sum_{\beta,\gamma=1}^{2} V_{\beta\gamma}\bar{\sigma}_{ij}^{(\beta\gamma)} \tag{8}$$

where $V = (h_1 + h_2)(l_1 + l_2)$. The resulting effective stress-strain equations for the composite obtained from the above analysis have the form:

$$\bar{\sigma} = C(\bar{\varepsilon} - \bar{\varepsilon}^p - \alpha\Delta T) \tag{9}$$

where the elements of the effective elastic stiffness tensor C, the elements of the effective thermal expansion coefficient α, and the effective plastic strains $\bar{\varepsilon}^p$ of the composite are expressed in terms of the elastic thermal properties and plastic strains of the constituent phases in *closed*

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form. These expressions are lengthy and thus will not be reproduced here - they can be found in References [1,2].

Although the method of cells has been demonstrated in numerous experimental/analytical correlation studies to be an accurate and, at the same time, efficient tool for analyzing the inelastic response of metal matrix composites in a wide temperature range, it suffers from several drawbacks. The rectangular geometry limits the analysis of MMC's to a limited number of fiber arrays. Only regular arrays with, at most, two different fiber spacings can be analyzed by the original method. The use of four subcells precludes the possibility of inclusion of an interfacial region between the fiber and the matrix phases, as well as consideration of more complicated fiber shapes. The transversely isotropic behavior of unidirectional composites in the plane perpendicular to the fiber direction is imposed artificially by setting $h_1 = l_1$ and $h_2 = l_2$ (see Figure 1), and subsequently averaging out the effect of the square geometry of the unit cell. These limitations motivated the development of the generalized version of the method of cells described subsequently.

Generalized method of cells

In the generalized formulation, the repeating unit cell is subdivided into an arbitrary number of subcells. This generalization extends the modelling capability of the method of cells to include the following:

- thermomechanical response of multi-phase, metal matrix composites
- modelling of variable fiber shapes
- analysis of different fiber arrays
- modelling of porosities and damage
- modelling of interfacial regions around inclusions, including interfacial degradation

The generalization of the original method of cells allows approximate micromechanical analysis of more complicated periodic arrays such as the one shown in Figure 2. The repeating volume element used in the GMC micromechanical analysis is shown in Figure 3. It consists of $N_\beta \times N_\gamma$ subcells. Each of these subcells is assumed to be occupied, in general, by an elastic-viscoplastic material. Thus the repeating volume element consists of $N_\beta \times N_\gamma$ different viscoplastic materials, i.e., it represents a multi-phase, inelastic composite.

A given elastic-viscoplastic phase is characterized in the elastic region by five elastic constants:

E_A = axial Young's modulus (sometimes called longitudinal or major)
E_T = transverse Young's modulus
ν_A = axial Poisson's ratio
ν_T = transverse Poisson's ratio
G_A = axial shear modulus

It is also characterized by two coefficients of thermal expansion:

α_A = axial CTE
α_T = transverse CTE

The form of the constitutive equation used to described the material behavior in the subcell $(\beta\gamma)$ in the generalized method of cells is precisely the same as for the original method of cells given by Equation (6). For a transversely isotropic material occupying the subcell $(\beta\gamma)$, the form of the elastic stiffness matrix in Equation (6) is:

$$C^{(\beta\gamma)} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{12} & C_{23} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}^{(\beta\gamma)} \tag{10}$$

where the x_1 direction is perpendicular to the plane of isotropy. The elastic stiffness matrix elements are related to the more commonly used engineering constants in the following manner:

$C_{11} = E_A + 4\kappa\nu_A^2$
$C_{12} = 2\kappa\nu_A$
$C_{22} = \kappa + G_T$
$C_{23} = \kappa - G_T$
$C_{44} = G_T$
$C_{55} = C_{66} = G_A$

where

$$G_T = \frac{1}{2}\frac{E_T}{1 + v_T} \qquad \text{and} \qquad \kappa = \frac{1}{4}\frac{E_A}{[\frac{1}{2}(1 - v_T)\frac{E_A}{E_T} - v_A^2]}.$$

The form of the thermal expansion coefficient matrix is:

$$\alpha^{(\beta\gamma)} = \text{diag}[\alpha_A^{(\beta\gamma)}, \alpha_T^{(\beta\gamma)}, \alpha_T^{(\beta\gamma)}, 0, 0, 0] \tag{11}$$

In the inelastic region the viscoplastic phase is represented at this time by the Bodner-Partom unified viscoplasticity theory. The local viscoplastic strain rate for a given inelastic sub-cell is expressed as

$$\dot{\epsilon}^{P\,(\beta\gamma)} = \Lambda_{(\beta\gamma)}\,\hat{s}^{(\beta\gamma)} \quad , \quad \beta + \gamma \neq 2 \tag{12}$$

where $\Lambda_{(\beta\gamma)}$ is the flow rule function of the inelastic phase and $\hat{s}^{(\beta\gamma)}$ are the stress deviators. The explicit form of the flow rule function is given by

$$\Lambda_{(\beta\gamma)} = D_0 \exp\{-\hat{n}\,[Z_{(\beta\gamma)}^2/(3J_2^{(\beta\gamma)})]^n\}\,/\,\sqrt{J_2^{(\beta\gamma)}} \tag{13}$$

where $\hat{n} = 0.5\,(n + 1)/n$, and $J_2^{(\beta\gamma)} = \frac{1}{2}\hat{s}^{(\beta\gamma)}\cdot\hat{s}^{(\beta\gamma)}$ is the second invariant of the average stresses in the inelastic subcell. D_0 and n are inelastic parameters, and $Z_{(\beta\gamma)}$ is a state variable given for an isotropic hardening material by

$$Z_{(\beta\gamma)} = Z_1^{(\beta\gamma)} + (Z_0^{(\beta\gamma)} - Z_1^{(\beta\gamma)})\exp[-m\,W_p^{(\beta\gamma)}/Z_0^{(\beta\gamma)}] \tag{14}$$

where $W_p^{(\beta\gamma)}$ is the plastic work per unit volume. For anisotropic (directional) hardening, the corresponding effective state variable is given by (see Reference [2, p.220]),

$$Z_{\text{eff}}^{(\beta\gamma)} = Z_0^{(\beta\gamma)} + q^{(\beta\gamma)}\int_0^t \dot{Z}_{(\beta\gamma)}d\tau + (1 - q^{(\beta\gamma)})\sum_{i,j=1}^{3}\int_0^t \dot{Z}_{(\beta\gamma)}r_{ij}d\tau \tag{15}$$

where $r_{ij} = \sigma_{ij}/\sqrt{\sigma_{kl}\sigma_{kl}}$, and $\dot{Z}_{(\beta\gamma)}$ is the time rate of change of the state variable for isotropic

hardening given by Equation (14).

The meaning of the six parameters D_0, Z_0, Z_1, n, m and q in Equations (14) and (15) is given below.

D_0 = limiting strain rate in shear for large values of the second stress invariant J_2

Z_0 = initial value of the hardening variable Z(t) which is related to the yield stress of the material in simple tension

Z_1 = saturation value of the hardening variable for large values of stresses

m = a parameter that controls the rate of work-hardening of the material

n = a parameter that controls the rate sensitivity of the material

q = a parameter that determines the relative weight of isotropic and directional hardening of the material. q = 1 provides a fully isotropic hardening and q = 0 corresponds to fully directional hardening

More information regarding the meaning and physical interpretation of these parameters can be found in Reference [2; pp. 216-224].

The micromechanical analysis of the repeating volume element in the generalized method of cells proceeds in exactly the same manner as that for the original method. In this case however, imposition of continuity of displacements and tractions between the individual subcells of the repeating unit cell, and between the given repeating cell and the surrounding cells, results in a system of equations which has a different appearance than that given by Equations (3)-(5). In fact, the generalization of the method of cells to an arbitrary number of subcells within the given repeating unit cell increases the complexity of the equations obtained by imposing the aforementioned continuity conditions and requires the use of compact notation. The system of equations relating the microvariables to the uniformly applied composite strains, obtained from the imposition of the continuity conditions, can be written in compact matrix notation as follows:

$$\tilde{A}\varepsilon_s - \tilde{A}^P \varepsilon_s^P = K\overline{\varepsilon} \tag{16}$$

where $\tilde{A}$, $\tilde{A}^P$, and K are appropriate matrices of coefficients (for temperature-independent properties, the elements of these matrices are constant); ε_s, ε_s^P are the total and plastic strain vectors for all the subcells; and $\overline{\varepsilon}$ are the average composite strains. The above equations can be solved in order to express the subcell strains in terms of the average strains:

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$$\varepsilon_s = A\bar{\varepsilon} + A^P \varepsilon_s^P \tag{17}$$

where $A = \tilde{A}^{-1} K$ and $A^P = \tilde{A}^{-1} \tilde{A}^P$. The matrix A, in fact, is the resulting Hill's concentration matrix which relates the micro to macro variables.

Using the definition of average stress in the repeating unit cell of the composite,

$$\bar{\sigma} = \frac{1}{hl} \sum_{\beta,\gamma} h_\beta l_\gamma \bar{\sigma}^{(\beta\gamma)} \tag{18}$$

we can readily establish the overall constitutive equations for the multi-phase, inelastic composite in the form:

$$\bar{\sigma} = B^* (\bar{\varepsilon} - \bar{\varepsilon}^P - \alpha^* \Delta T) \tag{19}$$

where

$\bar{\sigma}$ = the average composite stress

$\bar{\varepsilon}$ = the average composite total strain

$\bar{\varepsilon}^P$ = the average composite plastic strain

B^* = the effective elastic stiffness matrix of the composite

α^* = the effective CTE of the composite

The effective stiffness matrix of the composite, B^*, has the form,

$$B^* = \begin{bmatrix} b_{11} & b_{12} & b_{13} & 0 & 0 & 0 \\ b_{12} & b_{22} & b_{23} & 0 & 0 & 0 \\ b_{13} & b_{23} & b_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{66} \end{bmatrix}^* \tag{20}$$

Note that in the most general case, B^* represents a composite material whose effective behavior

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is orthotropic, in contrast to the transversely isotropic behavior given by Equation (9).

The effective engineering constants of the composite can be determined by inverting $\mathbf{B}^*$. This inversion produces the effective compliance matrix $\mathbf{S}^*$ whose elements are related to the engineering constants as follows:

$$E_{11}^* = 1/S_{11}^*$$

$$v_{12}^* = -S_{12}^*/S_{11}^*$$

$$E_{22}^* = 1/S_{22}^*$$

$$v_{23}^* = -S_{23}^*/S_{22}^*$$

$$E_{33}^* = 1/S_{33}^*$$

$$G_{23}^* = 1/S_{44}^*$$

$$G_{13}^* = 1/S_{55}^*$$

$$G_{12}^* = 1/S_{66}^*$$

The matrix of the effective coefficients of thermal expansion is given by

$$\alpha^* = \text{diag}[\alpha_{11}^*, \alpha_{22}^*, \alpha_{33}^*, 0, 0, 0] \qquad (21)$$

where α_{11}^*, α_{22}^* and α_{33}^* are the effective CTE's in the x_1, x_2 and x_3 directions, respectively.

The composite plastic strain vector $\overline{\epsilon}^p$ is determined by an incremental procedure in a step-wise manner. It is given in terms of the plastic strains in the individual subcells by an explicit expression which will not be reproduced here. Details of the GMC analysis can be found in Reference [3].

PROGRAM DESCRIPTION

The flow chart outlining the logical organization of the program **gmc.f** based on the generalized method of cells is given on the following page. The current capabilities of the program are listed in Table I.

The input data is read in from a file called **gmc.data** and the output is written to two files, namely **gmc.out** and **gmc.plot**. The input data is logically organized into four blocks, namely: the properties of different materials that occupy various subcells in the repeating unit cell; the loading applied to the composite; the internal microstructure of the repeating unit cell; and the geometrical details of the repeating unit cell. The output file **gmc.out** contains the echo of the input data, results for the effective thermoelastic properties, including elements of the effective stiffness matrix and the engineering properties, and the values of all the average stress and strain quantities calculated at the specified loading increments.

The program contains three parameter variables that can be reset within the program if desired. The program must be recompiled if these parameters are reset by the user. The three parameters are:

NBM = maximum number of subcells in the x_2 direction (set to 8)
NGM = maximum number of subcells in the x_3 direction (set to 8)
NMM = maximum number of different materials within the repeating unit cell (set to 5)

In the input file, the corresponding **actual** numbers for the user specified geometry are denoted by:

NB = actual number of subcells in the x_2 direction
NG = actual number of subcells in the x_3 direction
NMT = actual number of different materials

These numbers should not exceed the corresponding maximum numbers.

Detailed description of the input and output files, together with examples that illustrate the structure of these files and the capabilities of the program, are given on the following pages.

FLOW CHART FOR THE COMPUTER PROGRAM GMC.F

READ INPUT FILE GCM.DATA

material parameters, loading parameters,

RVE geometry

↓

GCM MICROMECHANICAL ANALYSIS

↓

WRITE GCM.OUT

effective thermoelastic coefficients

→ IF NINA = 0

yes

STOP

↓

WRITE GCM.OUT

inelastic response at every NSTEP increment

↓

WRITE GCM.PLOT

stress-strain response at every
NSTEP increment
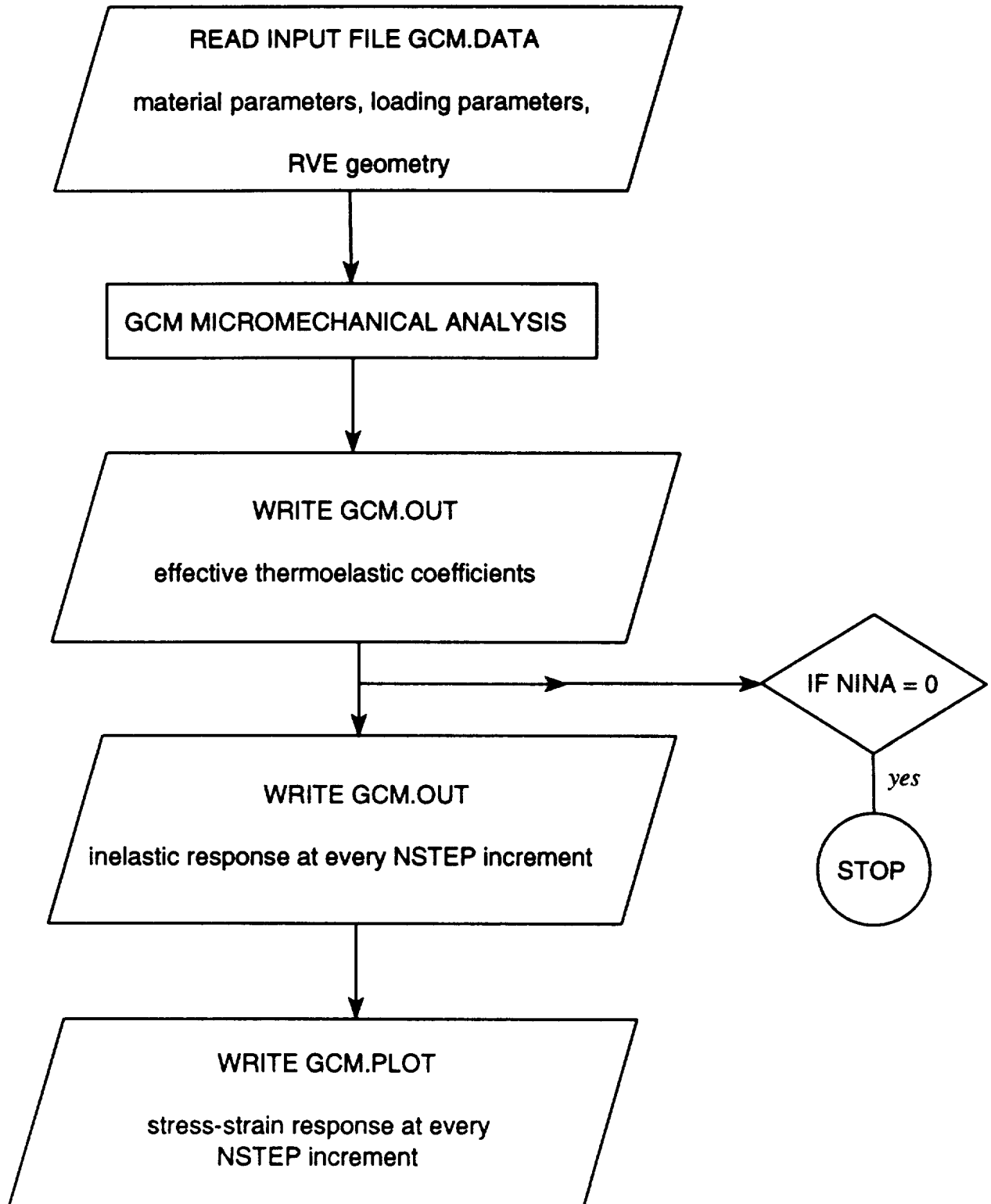
Table I. Current available capabilities within **GMC**.

Type	Description
Constitutive models	**Elastic**: isotropic, transversely isotropic materials **Viscoplastic**: Bodner-Partom unified theory for isotropic materials with isotropic or anisotropic hardening.
Integration schemes	Forward Euler integration technique
Loadings	Monotonic (strain control) 9 types of uniaxial load histories (see Table II)
Repeating unit cell geometry	6 standard geometries (see Table III) and a user specified geometry
Modelling capabilities	monotonic thermo-mechanical response modelling of variable fiber shapes analysis of different fiber arrays modelling of porosities and damage modelling of interface regions around inclusions and fibers
Predictive capabilities	effective (macroscopic) elastic moduli effective coefficients of thermal expansion effective thermal conductivities effective stress-strain response in the inelastic region

Input file gmc.data

The input file data is organized into four distinct blocks. The organization of the input data file and the variable names read by the program are given below.

Block 1 - Defines constituent material parameters

	NMT	- # of different materials
Repeat NMT times	MNAME	- material name
	EA,ET,GA	- elastic Young's and shear moduli
	FNA,FNT	- axial and transverse Poisson's ratios
	ALPA,ALPT	- axial and transverse thermal expansion coefficients
	D0,Z0,Z1	- Bodner-Partom viscoplastic model parameters[2]
	BM,AN,Q	- Bodner-Partom viscoplastic model parameters

Block 2 - Defines loading parameters[3]

	AMP,RATE	- final strain amplitude, strain rate of loading
	LOP,NINT,NINA	- load option (see Table II), number of load increments, actual number of load increments
5 lines	NSTEP	- output intervals
	THETA	- angle for LOP = 9 (skip if LOP is other than 9)
	DTEMP	- temperature deviation from reference temperature

Block 3 - Defines microstructure (assigns materials to subcells in the repeating unit cell).

	NB,NG	- number of subcells in the repeating unit cell in the x_2 and x_3 directions.
Repeat NG times	X...NB	- elements of the matrix MATNUM(NG,NB) containing material numbers corresponding to the material occupying the given subcell, starting with the subcell in the upper left corner of the repeating unit cell and progressing through NB columns (see Example 1). Each element of MATNUM specifies the material number MNAME(NM) in the given subcell ($\beta\gamma$), see Table III

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[2]Note that the material in the given subcell is perfectly elastic if D0 = 0.0.

[3] See footnotes a, b, and c on following page

Block 4 - Defines subcell geometry

ID — standard geometry definition (see Table III). The following are currently available: 1,2,3,4,5,6 or 10

VF — volume fraction (needed unless ID = 10). If ID = 10, the internal cell geometry must be input through the specification of the dimension of the individual subcells. In this case, we need to specify h_β and l_γ for $\beta = 1, \cdots, N_\beta$ and $\gamma = 1, \cdots, N_\gamma$, Figure 3, as follows:

NB lines
$\left\{ \begin{array}{l} h_\beta \\ h_{\beta-1} \\ \cdot \\ \cdot \\ \cdot \\ h_1 \end{array} \right.$

1 line $l_1 \quad l_2 \quad \dots \quad l_\gamma$

Footnotes

a.)

AMP is the final value (amplitude) of applied (monotonic) loading. For example, if we want to run the program in order to generate the response of the composite in the x_2 direction, then a stress-strain curve $\bar{\sigma}_{22} - \bar{\epsilon}_{22}$ is generated which gives the average composite stress $\bar{\sigma}_{22}$ for the given average composite strain $\bar{\epsilon}_{22}$. For a final value of $\bar{\epsilon}_{22}$ =0.02, say, AMP is input as 0.02. RATE is the rate of straining which is imposed in generating the $\bar{\sigma}_{22} - \bar{\epsilon}_{22}$ curve. For the rate of 0.01/sec, RATE is input as 0.01.

b.)

NINT is chosen such that the incremental integration procedure for the visco-plastic model converges. Since the response of an inelastic composite is generated by an incremental procedure, according to which the strain is changed in a step-wise manner, we need to define the number of increments required to achieve the final value of the applied strain (i.e., to reach AMP). This number is given by NINT. In case of nonconvergence, the user has to increase NINT. At this stage of the program development, nonconvergence is ascertained by the generation of very large values for the streses and strains. An automatic nonconvergence criterion will be incorporated into the program in the next phase of program development.

c.)

NINA is the actual number of increments that the program will execute and may be different from NINT. If the effective elastic moduli and coefficients of thermal expansion are the only quantities sought by the user, set NINA = 0.

Table II. Currently available loading options.

LOP Number	Description
LOP = 1	applies $\bar{\varepsilon}_{11}$ for a nonzero $\bar{\sigma}_{11}$ while keeping all other stresses zero (simulates uniaxial loading in the x_1 direction)
LOP = 2	applies $\bar{\varepsilon}_{22}$ for a nonzero $\bar{\sigma}_{22}$ while keeping all other stresses zero (simulates uniaxial loading in the x_2 direction)
LOP = 3	applies $\bar{\varepsilon}_{33}$ for a nonzero $\bar{\sigma}_{33}$ while keeping all other stresses zero (simulates uniaxial loading in the x_3 direction)
LOP = 4	applies $\bar{\varepsilon}_{23}$ for a nonzero $\bar{\sigma}_{23}$ while keeping all other stresses zero (simulates transverse shear loading in the $x_2 - x_3$ plane)
LOP = 5	applies $\bar{\varepsilon}_{13}$ for a nonzero $\bar{\sigma}_{13}$ while keeping all other stresses zero (simulates axial shear loading in the $x_1 - x_3$ plane)
LOP = 6	applies $\bar{\varepsilon}_{12}$ for a nonzero $\bar{\sigma}_{12}$ while keeping all other stresses zero (simulates axial shear loading in the $x_1 - x_2$ plane)
LOP = 7	applies $\bar{\varepsilon}_{22} = -\bar{\varepsilon}_{33}$ for nonzero $\bar{\sigma}_{22}$ and $\bar{\sigma}_{33}$ while keeping all other stresses zero (simulates pure shear in the $x_2 - x_3$ plane at 45 ° to the applied loading)
LOP = 8	applies ΔT (temperature deviation from a reference temperature) while keeping all composite stresses $\bar{\sigma}_{ij}$ zero.
LOP = 9	applies ε_{xx} to a unidirectional lamina in which the fibers are inclined at an angle θ with respect to the loading direction for a nonzero $\bar{\sigma}_{xx}$, while keeping all other stresses zero (simulates axial loading of an off-axis lamina).

Table III. Standard repeating unit cell geometry definitions (see Appendix I for a more detailed explanation).

ID Number	Geometry configuration	Material assignment matrix[4]
ID = 1	"square" fiber in a square array (see Figure 6)	2 2 1 2
ID = 2	"square" fiber in a triangular array (see Figure 7) VF < 0.288675 (see Figure 8a)	2 2 1 2 2 2 2 2 1 2 2 2 2 2 2 2
	VF > 0.288675 (see Figure 8b)	1 2 1 2 1 2 2 2 1 2 1 2 2 2 1 2
ID = 3	"square" fiber in a square diagonal array (see Figures 9 and 10)	2 2 1 2 2 2 2 2 1 2 2 2 2 2 2 2
ID = 4	cross-shaped fiber in a square array (see Figures 11 and 12)	2 2 2 2 2 1 2 2 1 1 1 2 2 1 2 2
ID = 5	coated, "square" fiber in a square array (see Figures 13 and 14)	2 2 2 2 3 3 3 2 3 1 3 2 3 3 3 2
ID = 6	circular fiber in a square array (see Figure 15)	2 2 2 2 2 2 2 2 2 1 2 2 2 2 2 1 1 1 2 2 2 1 1 1 1 1 2 2 2 1 1 1 2 2 2 2 2 1 2 2 2 2 2 2 2 2 2 2
ID = 10	user specified fiber shape and geometry	any assignment

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[4] Material 1 - fiber, material 2 - matrix, material 3 - coating

Example 1

This example illustrates the construction of the matrix MATNUM(NB,NG) that stores information on the types of materials that occupy various subcells. Each of the elements of MATNUM specifies the material number MNAME(NM) in the given subcell $(\beta\gamma)$. Let NMT = 2, NB = 3, NG = 4. For the specified NB and NG, the repeating unit cell has the form shown in Figure 4. In order to show the form of MATNUM(NB,NG), let us assign in this example 'Material 1' to the subcells (1,1), (2,2) and (1,4) while allowing the remaining subcells to be occupied by 'Material 2'. The resulting assignment is depicted in Figure 5.

Example 2

This example shows how to construct an input file in order to generate the transverse stress-strain response curve, $\bar{\sigma}_{22}$ versus $\bar{\varepsilon}_{22}$ (i.e. LOP = 2), of a boron/aluminum composite using a repeating unit cell that consists of 4 subcells (i.e. NB = 2 and NG = 2, ID = 1). The properties of the elastic boron fibers, 'Material 1', and the elastic-perfectly plastic aluminum matrix, 'Material 2', are given as taken from Reference [2], p. 248. They are reproduced in the table below.

Table IV. Material parameters of boron fibers and aluminum matrix

Material	E (GPa)	v	D_0^{-1} (sec)	Z_0 (MPa)	Z_1 (MPA)	m	n
Boron	413.70	0.20	-	-	-	-	-
Aluminum[5]	55.16	0.30	10^{-4}	103.42	103.42	-	10

The input file is given in Appendix II. Note that AMP = 0.005 and RATE = 0.01/sec. The number of increments required to generate the response curve is NINT = 2000. This number ensures that the resulting integration time increment is sufficiently small to attain convergence. The output is written to the output file gmc.out every 20 increments (since NSTEP = 20). In addition, DTEMP = 0 (i.e. $\Delta T = 0.0$) and VF=0.5.

---

[5]Note that since $Z_0 = Z_1$, the material is elastic, prefectly plastic and thus m is irrelevant (i.e., can be anything).

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Example 3

For the same boron/aluminum system of Example 2 we generate the transverse stress-strain response curve, $\bar{\sigma}_{22}$ versus $\bar{\varepsilon}_{22}$, using 4 x 4 subcells in the repeating unit cell. As the fundamental configuration and geometry of the repeating unit cell shown in Figure 16 produces the same doubly periodic array as in Example 2, the resulting stress-strain response curve should be identical. Here, NB = 4, NG = 4 and VF = 0.5.

Since the geometry in this case is supplied by the user, ID = 10 and so the dimensions h_β and l_γ of the subcells ($\beta\gamma$) must be specified. In order to generate VF = 0.5, we take

$h_1 + h_2 = 0.5$; $l_1 + l_2 = 0.5$
$h_1 = h_3$; $l_1 = l_3$
$h_2 = h_4$; $l_2 = l_4$
$h_1 = l_1$: $h_2 = l_2$

Since $VF = \dfrac{4 \times h_1^2}{1}$ -> $h_1 = \dfrac{1}{8} = 0.3535$. The subcell dimensions are input in the following manner

h_4
h_3
h_2
h_1
$l_1 \quad l_2 \quad l_3 \quad l_4$

or

0.1464
0.3535
0.1464
0.3535
0.3535 0.1464 0.3535 0.1464

The complete form of the input file for the present example is given in Appendix III.

Output file gmc.out

The first section of this file contains an "echo" of the input data that was read by the program from the input file **gmc.data**. The next section of the output file contains information about the subcell identification, material assignment in each subcell, volume of each subcell and the total volume of the repeating unit cell. It is a concise summary about the internal arrangement of the individual subcells.

The next section presents the effective stiffness matrix $\mathbf{B}^*$ of the composite. This is followed by the effective engineering elastic moduli:

E11S --> E_{11}^*

N12S --> ν_{12}^*

E22S --> E_{22}^*

N23S --> ν_{23}^*

E33S --> E_{33}^*

G23S --> G_{23}^*

G13S --> G_{13}^*

G12S --> G_{12}^*

The effective coefficients of thermal expansion are printed as follows:

ALPHA11S --> α_{11}^*

ALPHA22S --> α_{22}^*

ALPHA33S --> α_{33}^*

Note 1 : If the effective elastic moduli and coefficients of thermal expansion are the only quantities sought by the user, the parameter NINA = 0 in the input file as mentioned previously. In this case, the program will not generate the inelastic response and the output file will terminate at this stage.

Note 2 : If the effective thermal conductivities of the composite k_{11}^*, k_{22}^* and k_{33}^*, are sought, the user should set NINA = 0, and the following replacement should be made in the input file:

EA(NM) --> k_A for each material type

ET(NM) --> k_T for each material type

FNA(NM) = FNT(NM) = 0

GA(NM) is arbitrary

where k_A and k_T are the axial and transverse thermal conductivities of a transversely isotropic matarial which occupies a given subcell. In the output file, k_{11}^*, k_{22}^* and k_{33}^* are identified by E_{11}^*, E_{22}^* and E_{33}^*, respectively.

The next section presents the inelastic response when the parameter *NINA* is greater than zero. The program prints the following results at the first increment (*INC* = *1*) and subsequently at every *NSTEP* increments according to the format:

INC = **increment number**

IJ component	Total strains	Stresses	Plastic strains	Thermal strains
11	...	...	...	...
22	...	...	...	...
33	...	...	...	...
23	...	...	...	...
13	...	...	...	...
12	...	...	...	...

The program is terminated when the increment number *INC* reaches *NINA*. A sample **gmc.out** output file is presented in Appendix IV generated with the input file **gmc.data** of Example 3.

Output file gmc.plot

For the parameter *NINA* greater or equal to 1, the program writes to this file the specified stress-strain response dictated by the loading option *LOP*, starting at the first increment *INC = 1*, and at each subsequent *NSTEP* increment. The loading option specifies the stress-strain response written to this file according to;

$LOP = 1$ $\bar{\varepsilon}_{11}$ $\bar{\sigma}_{11}$

$LOP = 2$ $\bar{\varepsilon}_{22}$ $\bar{\sigma}_{22}$

$LOP = 3$ $\bar{\varepsilon}_{33}$ $\bar{\sigma}_{33}$

$LOP = 4$ $\bar{\varepsilon}_{23}$ $\bar{\sigma}_{23}$

$LOP = 5$ $\bar{\varepsilon}_{13}$ $\bar{\sigma}_{13}$

$LOP = 6$ $\bar{\varepsilon}_{12}$ $\bar{\sigma}_{12}$

$LOP = 7$ $\dfrac{1}{2}(\bar{\varepsilon}_{22} - \bar{\varepsilon}_{33})$ $\dfrac{1}{2}(\bar{\sigma}_{22} - \bar{\sigma}_{33})$

$LOP = 8$ ΔT $\bar{\varepsilon}_{11}$ $\bar{\varepsilon}_{22}$ $\bar{\varepsilon}_{33}$

$LOP = 9$ $\bar{\varepsilon}_{xx}$ $\bar{\sigma}_{xx}$

ACKNOWLEDGEMENTS

REFERENCES

1. Aboudi, J., "Micromechanical Analysis of Composites by the Method of Cells," **Applied Mechanics Reviews**, Vol. 42, pp. 193- 221, 1989.

2. Aboudi, J., **Mechanics of Composite Materials : A Unified Micromechanical Approach**, Elsevier, Amsterdam, 1991.

3. Paley, M. and Aboudi, J., "Micromechanical Analysis of Composites by the Generalized Cells Model", (in press).

APPENDIX I

A more detailed explanation of the standard repeating unit cell geometry definitions summarized in Table II are given in this appendix.

ID = 1 : corresponds to the original method of cells (NB = NG = 2) in which the fiber occupies subcell (1,1), with the other three subcells occupied by the matrix as shown in Figure 6. Let VF denote the fiber volume fraction. The dimension of the repeating unit cell are scaled to 1×1 so that $h_1 + h_2 = l_1 + l_2 = 1$. Thus, for a "square" fiber, specification of VF is sufficient to define the dimension h_β and l_γ, i.e., $h_1 = \sqrt{VF}$, $h_2 = 1 - h_1$, $l_1 = h_1$ and $l_2 = h_2$. The material assignment of the fiber (material 1) and matrix (material 2) is as follows:

$$\begin{array}{|cc|} \hline 2 & 2 \\ 1 & 2 \\ \hline \end{array}$$

ID = 2 : gives a triangular packing of "square" fibers shown in Figure 7. In this arrangement, NB = NG = 4, and for a given fiber volume fraction VF the dimensions h_β and l_γ are automatically determined internally. For this packing arrangement, it is necessary to differentiate between two cases which is done internally. For VF < 0.288675, the geometry of the repeating unit cell is given in Figure 8a, where

$h_1 = b - \dfrac{a}{2}$

$h_2 = a$

$h_3 = h_1$

$h_4 = a$

and

$l_1 = a$

$l_2 = c$

$l_3 = a$

$l_4 = c$

where

$a = \sqrt{\dfrac{1}{2}\sqrt{3}\,VF}$

$b = \dfrac{1-a}{2}$

$c = \sqrt{\dfrac{3}{4}} - a$

The material assignment of the fiber (material 1) and matrix (Material 2) in this case is:

$$\begin{array}{|cccc|}\hline 2 & 2 & 1 & 2 \\ 2 & 2 & 2 & 2 \\ 1 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\\hline\end{array}$$

For VF > 0.288675, the geometry of the repeating unit cell is given in Figure 8b, where

$h_1 = 2b$

$h_2 = \dfrac{a}{2} - b$

$h_3 = h_1$

$h_4 = h_2$

and

$l_1 = a$

$l_2 = c$

$l_3 = l_1$

$l_4 = l_2$

The material assignment of the fiber (material 1) and matrix (material 2) in this situation is:

$$\begin{array}{|cccc|}\hline 1 & 2 & 1 & 2 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 1 & 2 \\ 2 & 2 & 1 & 2 \\\hline\end{array}$$

ID = 3 : specifies a square diagonal packing illustrated in Figure 9. In this case NB = NG = 4. The subcell dimensions h_β and l_γ are determined internally in the program for the specified fiber volume fraction VF. For this packing arrangement, VF cannot exceed 0.5. The dimension of the repeating unit cell for this packing arrangement shown in Figure 10, where

$h_1 = \dfrac{1}{2}[1 - 2\sqrt{\dfrac{VF}{2}}]$

$h_2 = \sqrt{\dfrac{VF}{2}}$

$h_3 = h_1$

$h_4 = h_4$

and

$l_1 = h_2$
$l_2 = h_1$
$l_3 = l_1$
$l_4 = l_2$

The material assignment of the fiber (material 1) and matrix (material 2) for this case is:

$$\begin{array}{|cccc|} \hline 2 & 2 & 1 & 2 \\ 2 & 2 & 2 & 2 \\ 1 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ \hline \end{array}$$

ID = 4 : models a cross-shaped fiber shown in Figure 11. It requires that NB = NG = 4, and the parameter "a" shown in the figure must be provided by the user. Thus in this case the parameter "a" should be read by the statement READ A. For this arrangement, the geometric details of the repeating unit cell are shown in Figure 12. Defining $x = -2a + \sqrt{4a^2 + VF}$, *the dimensions* h_β *and* l_γ *are given by*

$h_1 = a$
$h_2 = x$
$h_3 = a$
$h_4 = 1 - 2a - x$

and

$l_1 = h_1$
$l_2 = h_2$
$l_3 = h_3$
$l_4 = h_4$

The material assignment of the fiber (material 1) and matrix (material 2) for this case is:

$$\begin{array}{|cccc|} \hline 2 & 2 & 2 & 2 \\ 2 & 1 & 2 & 2 \\ 1 & 1 & 1 & 2 \\ 2 & 1 & 2 & 2 \\ \hline \end{array}$$

ID = 5 : models a coated, "square" fiber shown in Figure 13. Let VF and VC denote, respectively, the fiber volume fraction and the coating volume fraction. This situation requires that NB = NG = 4. Note that in this case the parameter VC is read by the statement READ VC. The dimensions of the repeating unit cell for this arrangement are given in Figure 14, where

$$h_1 = \frac{1}{2}[-\sqrt{VF} + \sqrt{VF + VC}]$$
$$h_2 = \sqrt{VF}$$
$$h_3 = h_1$$
$$h_4 = 1 - [h_1 + h_2 + h_3]$$

and

$$l_1 = h_1$$
$$l_2 = h_2$$
$$l_3 = h_3$$
$$l_4 = h_4$$

The material assignment of the fiber (material 1), matrix (material 2), and coating (material 3) for this case is:

2	2	2	2
3	3	3	2
3	1	3	2
3	3	3	2

ID = 6 : this case approximates a circular fiber using $N_\beta = 7$ and $N_\gamma = 7$ subcells as shown in Figure 15. By imposing the condition that $v_f = \pi R^2/h^2$, where R is the fiber radius, it follows that

$$h_2 = h_3 = h_5 = h_6 = \sqrt{\frac{\pi}{52}}\, R$$
$$h_4 = 4h_2$$
$$h_1 = h_7 = [h - (2h_2 + 2h_3 + h_4)]/2 \ and \ l_i = h_i \ for \ i = 1, \cdots, 7$$

The material assignment of the fiber (material 1) and matrix (material 2) for this case is:

2	2	2	2	2	2	2
2	2	2	1	2	2	2
2	2	1	1	1	2	2
2	1	1	1	1	1	2
2	2	1	1	1	2	2
2	2	2	1	2	2	2
2	2	2	2	2	2	2

ID = 10 : enables the user to specify his own internal configuration by specifying the dimensions h_β and l_γ.

APPENDIX II

The input file for the case described in Example 2 is given below.

```
              2
              'Material 1'
              413.70E+09   413.70E+09   172.30E+09
                0.20E+00     0.20E+00
                1.00E-06     1.00E-06
                0.00E+00     1.00E+00     1.00E+00
                1.00E+00     1.00E+00     1.00E+00
Block 1       'Material 2'
                55.16E+09    55.16E+09    21.20E+09
                0.30E+00     0.30E+00
                1.00E-06     1.00E-06
                1.00E+04   103.42E+06   103.42E+06
                1.00E+00    10.00E+00     1.00E+00
              0.005   0.01
              2   2000   2000
Block 2       20
              0.0
              2 2
Block 3       2 2
              1 2
              1
Block 4       0.5
```

APPENDIX III

The input file for the case described in Example 3 is given below.

```
          2
          'Material 1'
          413.70E+09   413.70E+09   172.30E+09
            0.20E+00     0.20E+00
            1.00E-06     1.00E-06
            0.00E+00     1.00E+00      1.00F+00
            1.00E+00     1.00E+00      1.00E+00
          'Material 2'
           55.16E+09    55.160E+09   21.20E+09
            0.30E+00     0.30E+00
            1.00E-06     1.00E-06
            1.00E+04   103.42E+06   103.42E+06
            1.00E+00    10.00E+00      1.00E+00
```

Block 1 brackets the first material section (from 2 through the Material 2 data).

```
Block 2   {  0.005  0.01
             2  2000  2000
             20
             0.0
```

```
Block 3   {  4 4
             2 2 2 2
             1 2 1 2
             2 2 2 2
             1 2 1 2
```

```
Block 4   {  10
             0.1464
             0.3535
             0.1464
             0.3535
             0.3535   0.1464   0.3535   0.1464
```

APPENDIX IV

The output file **gmc.out** generated by the input file presented in Example 3 is given below.

```
*********************************************************
**              GENERALIZED CELLS MODEL              **
**                                                   **
**                    G C M                          **
**                                                   **
**   Determination of effective elastic, thermal     **
**     and inelastic response of unidirectional,     **
**    multi-phase composites using generalized       **
**          version of the method of cells           **
**                                                   **
**                 Programmed by                      **
**            Jacob Aboudi & Moshe Paley             **
**            NASA Lewis version - July 1992          **
*********************************************************
```

```
***** INPUT DATA ECHO *****
```

MATERIAL SPECIFICATION

Number of Materials (NMT) = 2

Material 1

 Thermo-elastic constants

EA   = 0.414E+12   ET   = 0.414E+12   GA = 0.172E+12
NUA = 0.200E+00   NUT = 0.200E+00
ALFA = 0.100E-05   ALFT = 0.100E-05

 Inelastic parameters

D0   = 0.000E+00   Z0   = 0.100E+01   Z1 = 0.100E+01
M   = 0.100E+01   N   = 0.100E+01   Q = 0.100E+01

Material 2

 Thermo-elastic constants

EA   = 0.552E+11   ET   = 0.552E+11   GA = 0.212E+11
NUA = 0.300E+00   NUT = 0.300E+00
ALFA = 0.100E-05   ALFT = 0.100E-05

 Inelastic parameters

D0   = 0.100E+05   Z0   = 0.103E+09   Z1 = 0.103E+09
M   = 0.100E+01   N   = 0.100E+02   Q = 0.100E+01

LOADING SPECIFICATION

Amplitude = 0.500E-02   Rate = 0.100E-01
Loading option =   2
Number of integration increments =   2000
Actual number of increments =   2000
Number of print steps =   20
Temperature change   = 0.000E+00

RVE SPECIFICATION

Number of subcells in the x-2 direction =   4
Number of subcells in the x-3 direction =   4

Subcell material assignment

   2 2 2 2
   1 2 1 2
   2 2 2 2
   1 2 1 2

Subcell dimension assignment

ID =   10

H 4   = 0.146E+00
H 3   = 0.354E+00
H 2   = 0.146E+00
H 1   = 0.354E+00

L(1,...,NG) = 0.354E+00 0.146E+00 0.354E+00 0.146E+00

***** OUTPUT RESULTS *****

SUBCELL IDENTIFICATION, VOLUME AND MATERIAL ARRANGEMENT

(BETA , GAMMA)	SUBCELL #	SUBCELL MATERIAL	SUBCELL VOLUME
1 , 1	1	1	0.125E+00
1 , 2	2	2	0.518E-01
1 , 3	3	1	0.125E+00
1 , 4	4	2	0.518E-01
2 , 1	5	2	0.518E-01
2 , 2	6	2	0.214E-01
2 , 3	7	2	0.518E-01
2 , 4	8	2	0.214E-01
3 , 1	9	1	0.125E+00
3 , 2	10	2	0.518E-01
3 , 3	11	1	0.125E+00
3 , 4	12	2	0.518E-01
4 , 1	13	2	0.518E-01
4 , 2	14	2	0.214E-01

```
4 , 3                15                    2                0.518E-01
4 , 4                16                    2                0.214E-01


                                         TOTAL VOLUME =  0.100E+01
```

Effective stiffness matrix

```
0.258E+12    0.473E+11    0.473E+11
0.473E+11    0.149E+12    0.446E+11
0.473E+11    0.446E+11    0.149E+12
                                      0.378E+11
                                                  0.457E+11
                                                              0.457E+11
```

Effective engineering moduli

```
E11S=  0.235E+12
N12S=  0.245E+00
E22S=  0.131E+12
N23S=  0.257E+00
E33S=  0.131E+12
G23S=  0.378E+11
G13S=  0.457E+11
G12S=  0.457E+11
```

Effective thermal expansion coefficients

```
ALPHA11S=  0.100E-05
ALPHA22S=  0.100E-05
ALPHA33S=  0.100E-05
```

EFFECTIVE STRESS-STRAIN RESPONSE

INC =   1

IJ	TOTAL STRAINS	STRESSES	PLASTIC STRAINS	THERMAL STRAINS
11	-0.341E-06	0.195E-02	0.000E+00	0.000E+00
22	0.250E-05	0.327E+06	0.000E+00	0.000E+00
33	-0.642E-06	0.391E-01	0.000E+00	0.000E+00
23	0.000E+00	0.000E+00	0.000E+00	0.000E+00
13	0.000E+00	0.000E+00	0.000E+00	0.000E+00
12	0.000E+00	0.000E+00	0.000E+00	0.000E+00

INC =   20

IJ	TOTAL STRAINS	STRESSES	PLASTIC STRAINS	THERMAL STRAINS	
					32

IJ	TOTAL STRAINS	STRESSES	PLASTIC STRAINS	THERMAL STRAINS
11	-0.682E-05	0.625E-01	-0.504E-21	0.000E+00
22	0.500E-04	0.654E+07	0.328E-20	0.000E+00
33	-0.128E-04	0.500E+00	-0.232E-20	0.000E+00
23	0.000E+00	0.000E+00	0.000E+00	0.000E+00
13	0.000E+00	0.000E+00	0.000E+00	0.000E+00
12	0.000E+00	0.000E+00	0.000E+00	0.000E+00

INC =   40

IJ	TOTAL STRAINS	STRESSES	PLASTIC STRAINS	THERMAL STRAINS
11	-0.136E-04	0.125E+00	-0.103E-20	0.000E+00
22	0.100E-03	0.131E+08	0.673E-20	0.000E+00
33	-0.257E-04	0.100E+01	-0.476E-20	0.000E+00
23	0.000E+00	0.000E+00	0.000E+00	0.000E+00
13	0.000E+00	0.000E+00	0.000E+00	0.000E+00
12	0.000E+00	0.000E+00	0.000E+00	0.000E+00

.
.
.
.
.
.
.
.

INC = 1980

IJ	TOTAL STRAINS	STRESSES	PLASTIC STRAINS	THERMAL STRAINS
11	-0.316E-03	-0.400E+01	-0.121E-03	0.000E+00
22	0.495E-02	0.187E+09	0.352E-02	0.000E+00
33	-0.355E-02	0.240E+02	-0.318E-02	0.000E+00
23	0.000E+00	0.000E+00	0.000E+00	0.000E+00
13	0.000E+00	0.000E+00	0.000E+00	0.000E+00
12	0.000E+00	0.000E+00	0.000E+00	0.000E+00

INC = 2000

IJ	TOTAL STRAINS	STRESSES	PLASTIC STRAINS	THERMAL STRAINS
11	-0.316E-03	-0.600E+01	-0.121E-03	0.000E+00
22	0.500E-02	0.187E+09	0.357E-02	0.000E+00
33	-0.360E-02	0.000E+00	-0.323E-02	0.000E+00
23	0.000E+00	0.000E+00	0.000E+00	0.000E+00
13	0.000E+00	0.000E+00	0.000E+00	0.000E+00
12	0.000E+00	0.000E+00	0.000E+00	0.000E+00

a) Doubly periodic array   b) Representative unit cell
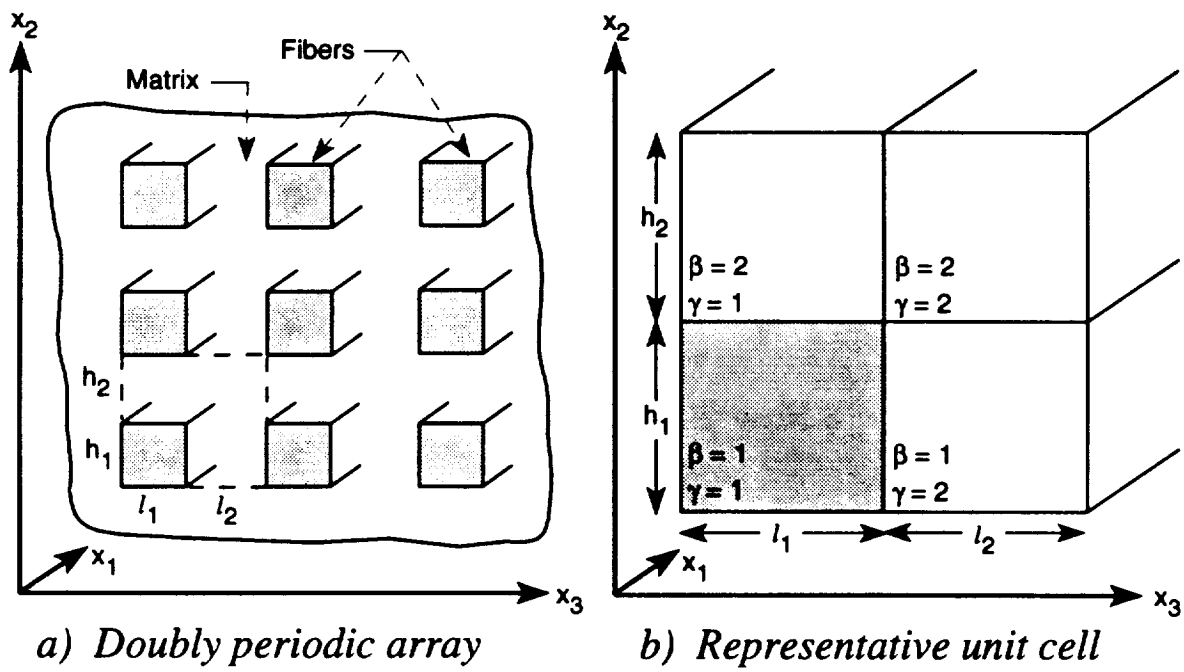
Figure 1. Doubly-periodic array and a repeating unit cell for the original method of cells.
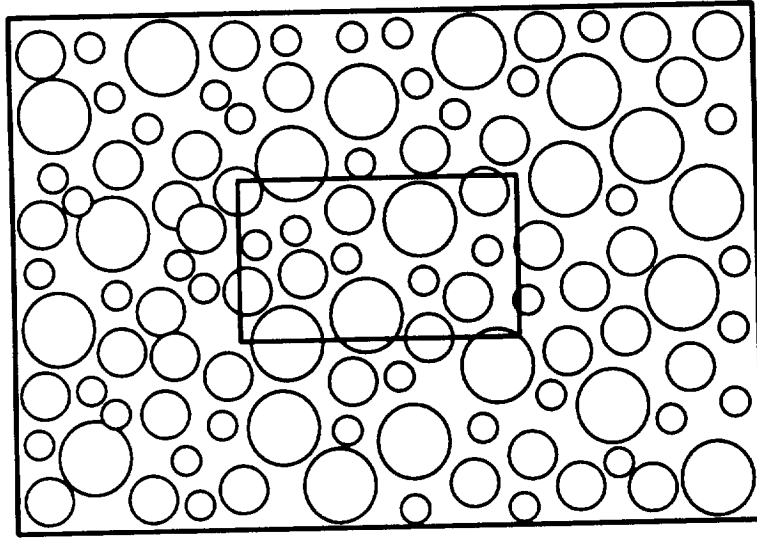
Figure 2. Unit periodic geometry for the generalized method of cells (GMC).
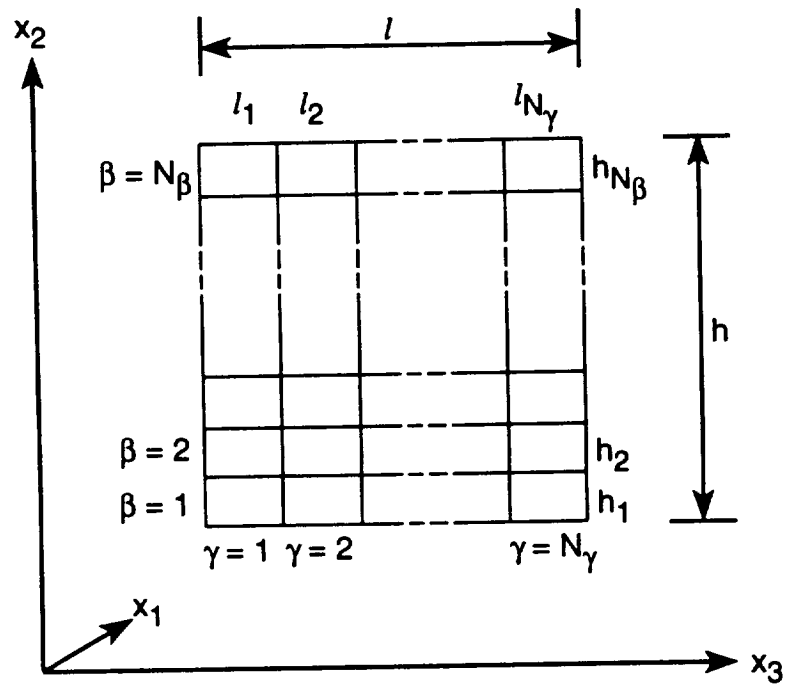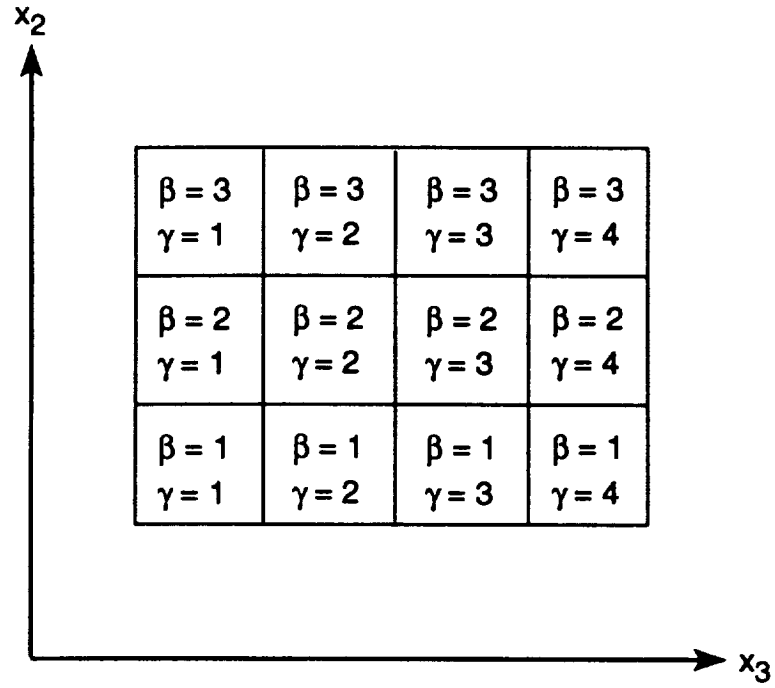

Figure 3. A repeating volume element of GMC.

Figure 4. A repeating volume element with NB = 3 and NG = 4 (Example 1).
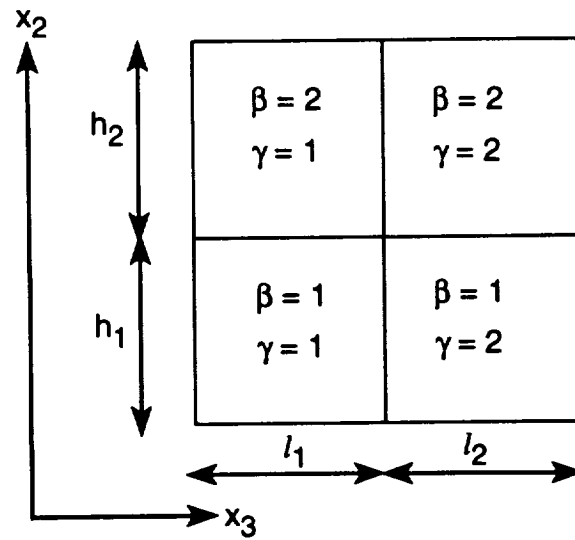

Figure 5. Material assignment for Figure 4.

Figure 6. Repeating volume element for the case ID = 1, with the fiber assigned to the subcell (1,1) and the matrix to the remaining subcells.
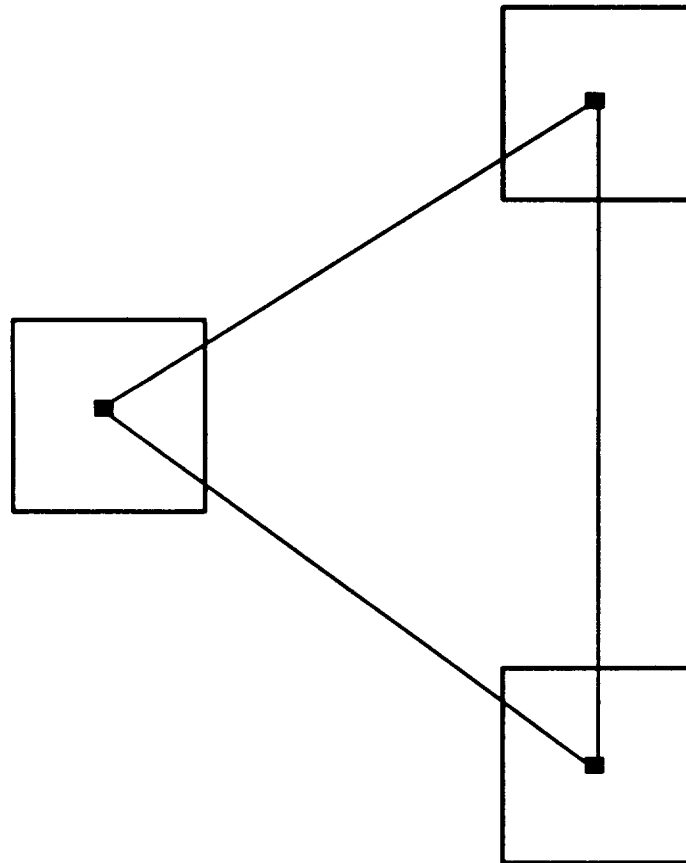
Figure 7. Fiber arrangement for triangular packing (ID = 2).
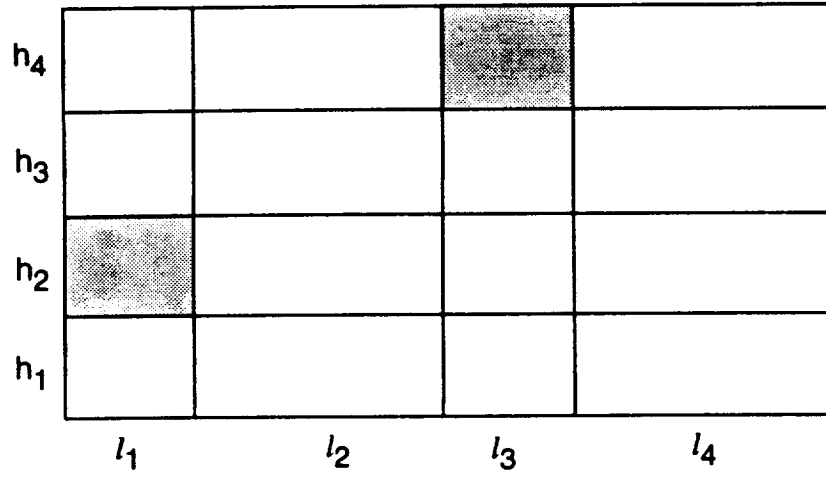
a.)
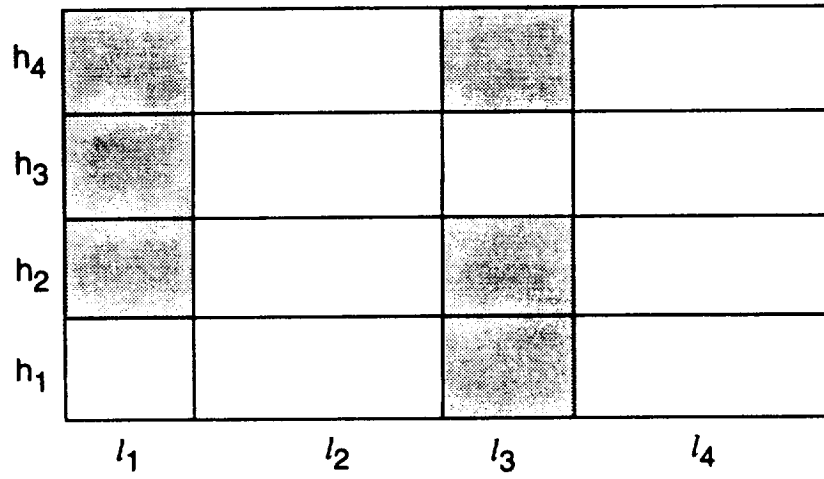
b.)

Figure 8. Fiber and matrix locations in a repeating unit cell for triangular packing (ID = 2) when a.) $v_f < 0.288675$ and b.) $v_f > 0.288675$.
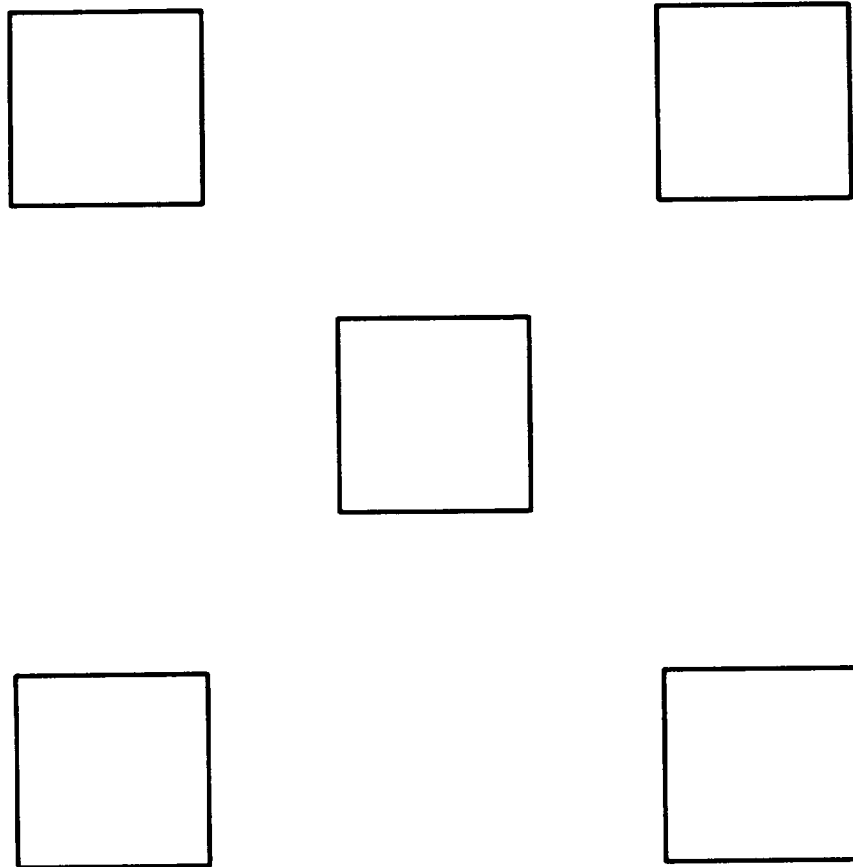
Figure 9. Fiber arrangement for square diagonal packing (ID = 3).
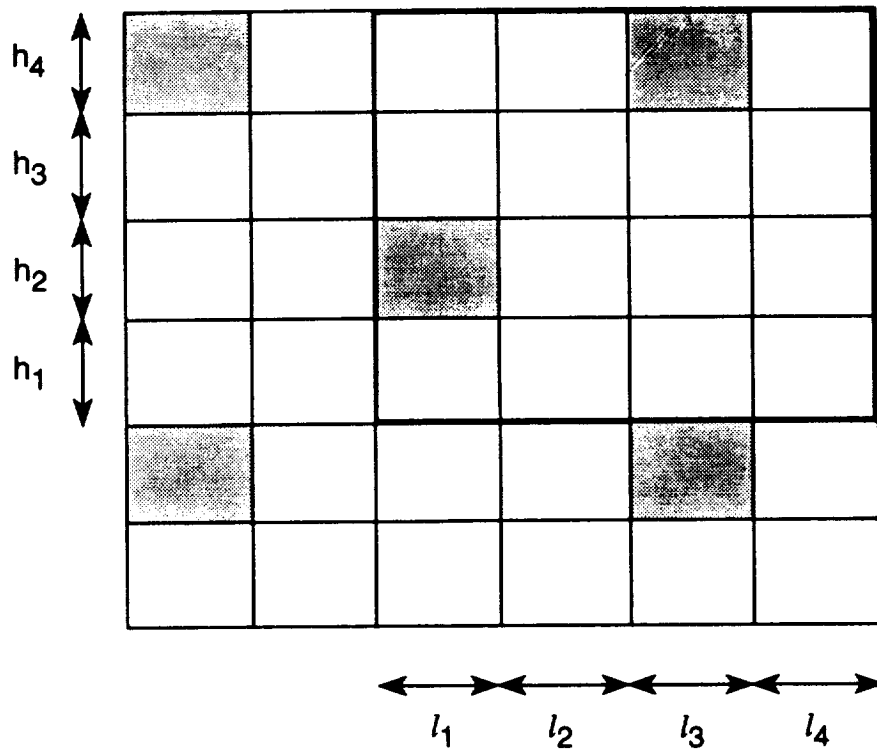
Figure 10. Fiber and matrix locations in a repeating unit cell for square diagonal packing (ID = 3).
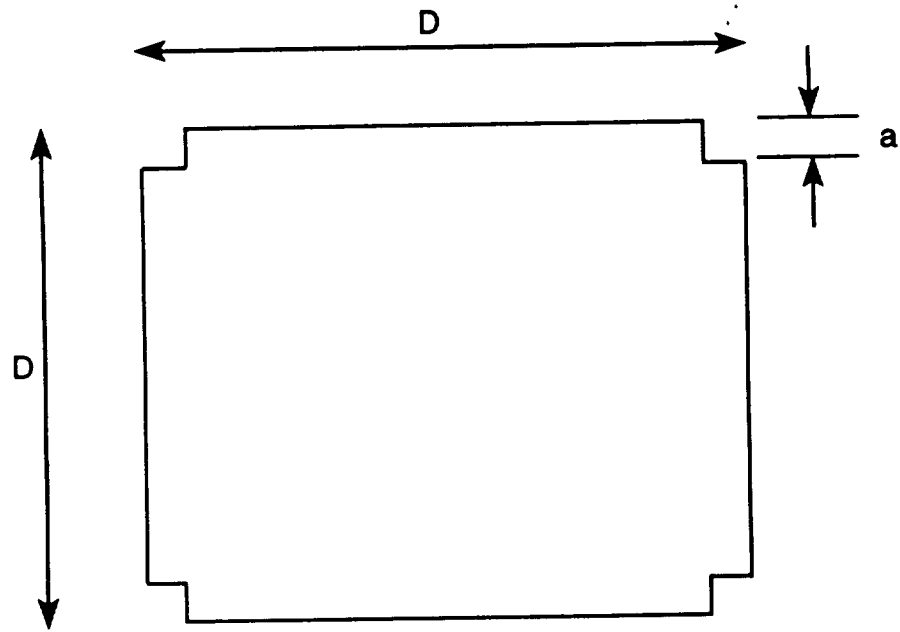
Figure 11. A cross-shaped fiber (ID = 4).
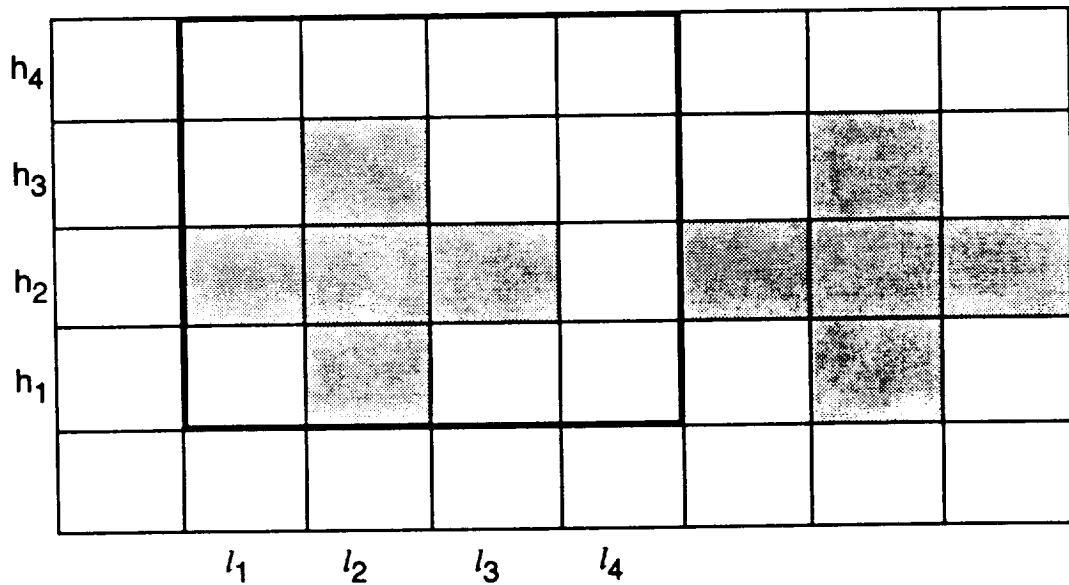
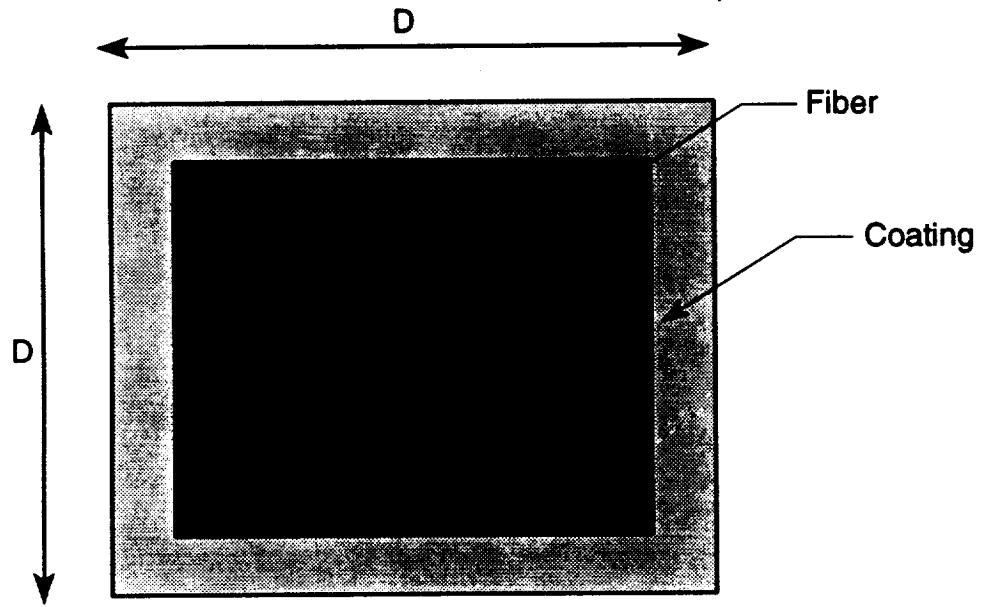Figure 12. Fiber and matrix locations in a repeating unit cell for a cross-shaped fiber (ID = 4).
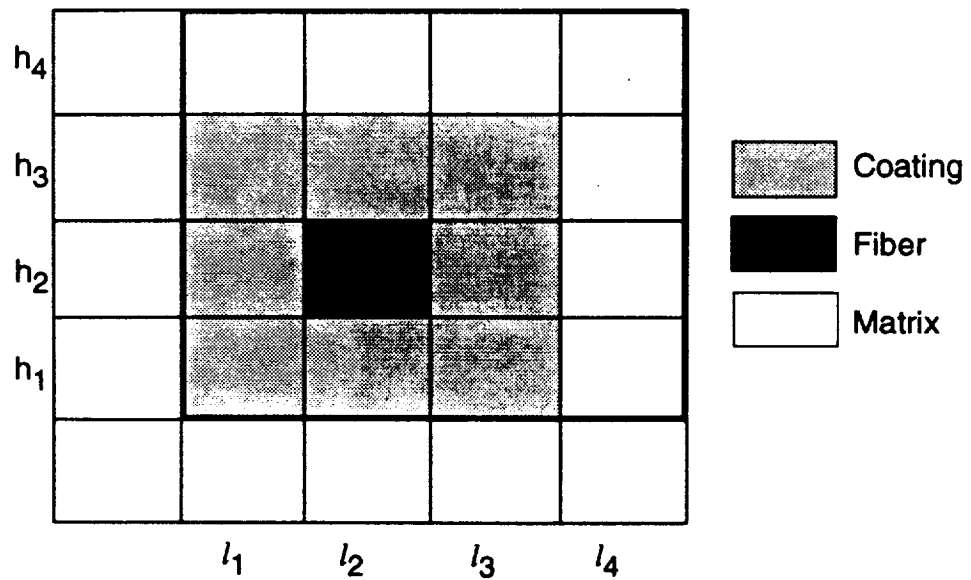
Figure 13. A fiber with a coating (ID = 5).

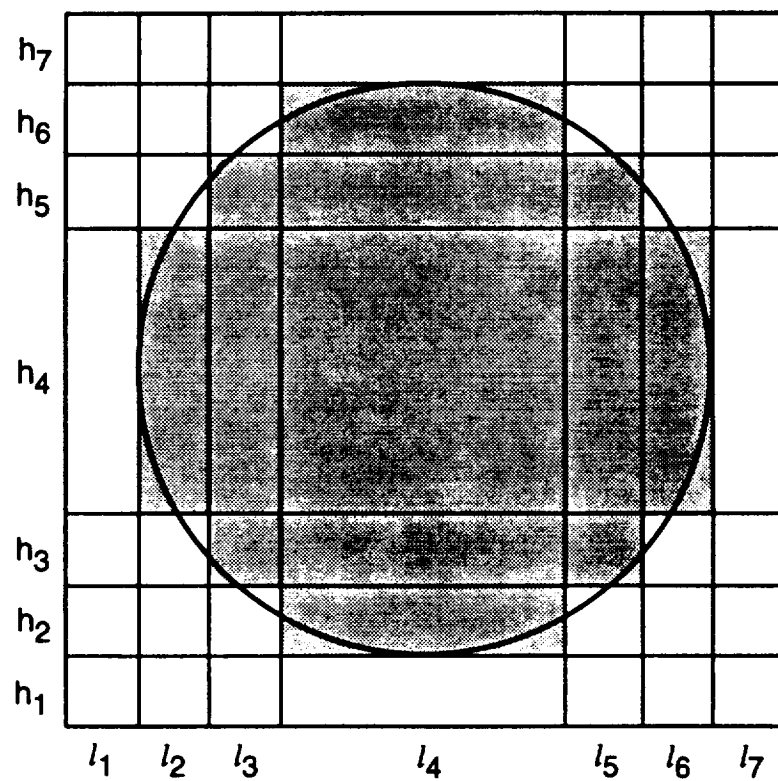Figure 14. Fiber, matrix and coating locations in a repeating unit cell (ID = 5).

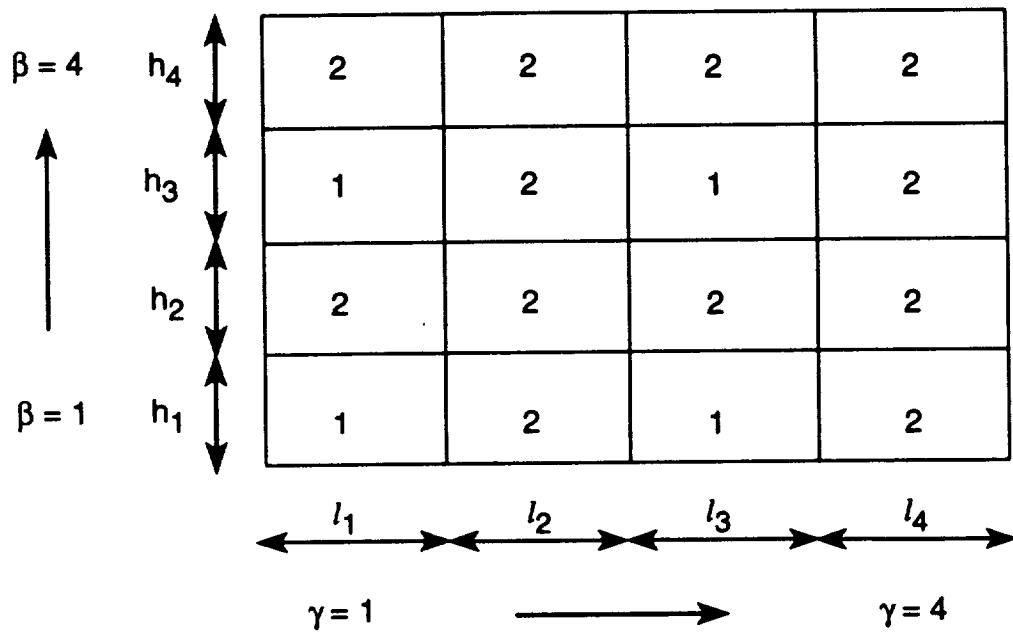Figure 15. Repeating volume element for a circular fiber approximation (ID = 6).

Figure 16. Material assignment for Example 3.